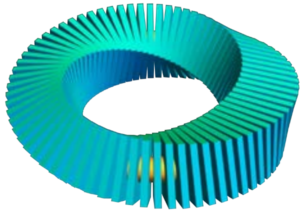
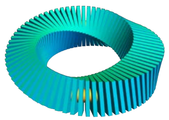
Winter School on Quantum Information Science (Jan 10-21, 2022)



Surface Codes

A Class of Stabilizer Codes with Topological Features



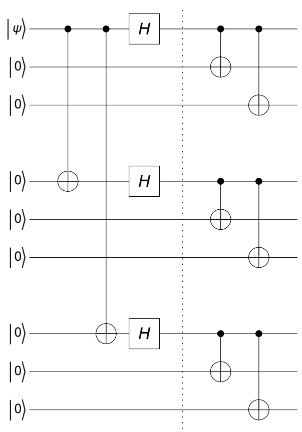
Mahn-Soo Choi
(Korea University)

Why Special?

Surface Codes

Rely only ***local*** operations and measurements.

9-Qubit Code

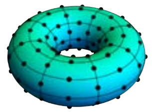


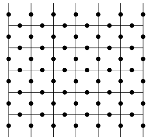
A Bit of History

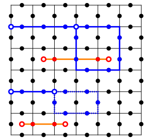
- A. Yu. Kitaev (Ann. Phy., 2003; quant-ph/9707021)
- Dennis, Kitaev, Landahn, & Preskill (J. Math. Phys., 2002)
- M. H. Freedman (Found. Comp. Math., 2001)
- Fowler, Mariantoni, Martinis, & Cleland (Phys. Rev. A, 2012)

OUTLINE

1. Toric Codes
   - Stabilizers
   - Local operators
   - Error syndrome detection and recovery
2. Planar Codes
   - Stabilizers
   - Local operators
   - Error syndrome detection and recovery
3. Topological Features



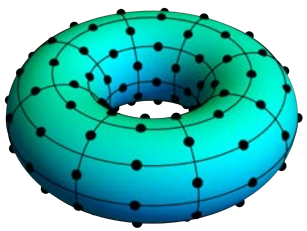


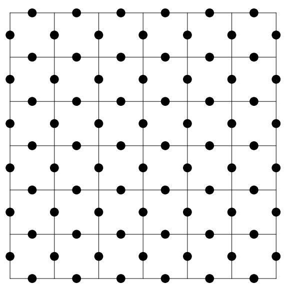


TORIC CODES

QUBITS ARRANGED ON THE SURFACE OF A TORUS

A SQUARE LATTICE WITH PERIODIC BOUNDARY CONDITIONS





STABILIZERS

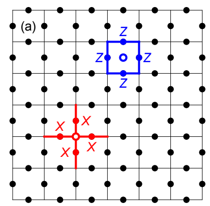
PLAQUETTE & VERTEX OPERATORS

- Plaquette operator

$$\hat{P}_p = \hat{Z}_{p,1}\hat{Z}_{p,2}\hat{Z}_{p,3}\hat{Z}_{p,4}$$

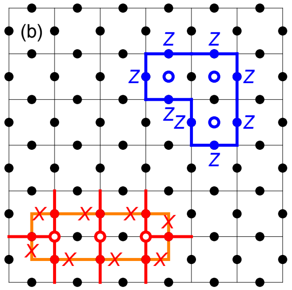
- Vertex operator

$$\hat{V}_v = \hat{X}_{v,1}\hat{X}_{v,2}\hat{X}_{v,3}\hat{X}_{v,4}$$



Multiplications of Stabilizers

Plaquette & Vertex Operators

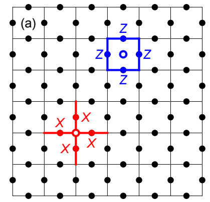


$$\prod_p \hat{P}_p = \prod_v \hat{V}_v = \hat{I}$$

How Many Stabilizers?

Dimension of the code space?

- There are L^2 plaquette operators, but they must satisfy one constraint.
- There are L^2 vertex operators, but they must satisfy one constraint.
- Overall, there are $2L^2$-2 stabilizers.
- There are $2L^2$ qubits (edges).
- The code can encode two logical qubits.

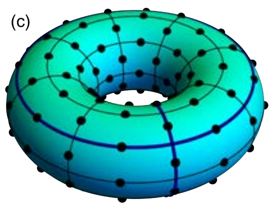


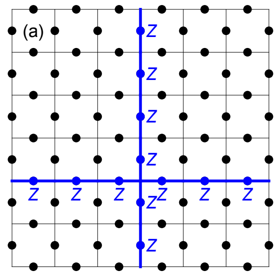
Where are two more compatible operators?

Where are two logical Z operators?

Where are two more compatible operators?

Where are two logical Z operators?



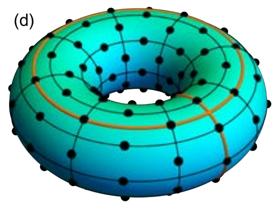


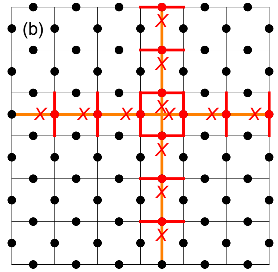
Where are two more compatible operators?

Where are two logical X operators?

Where are two more compatible operators?

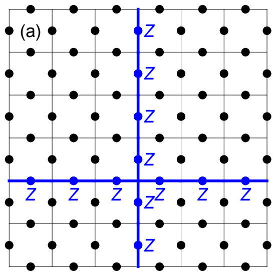
Where are two logical X operators?

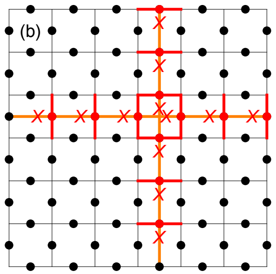




Check!

Do they anti-commute?

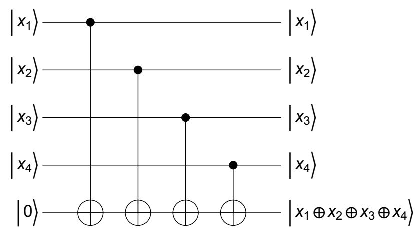


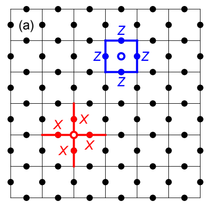


Error Syndrome Measurements

Plaquette Operators

$$\hat{P}_p = \hat{Z}_{p,1}\hat{Z}_{p,2}\hat{Z}_{p,3}\hat{Z}_{p,4}$$

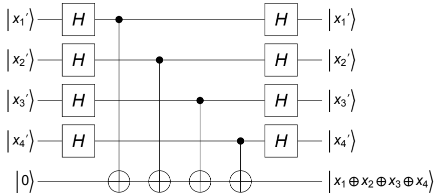


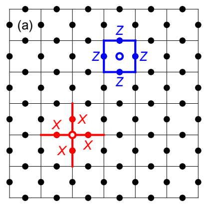


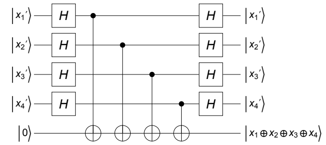
ERROR SYNDROME MEASUREMENTS

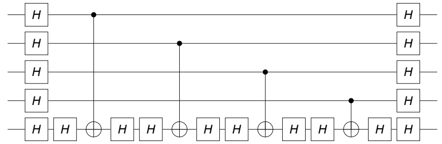
VERTEX OPERATORS

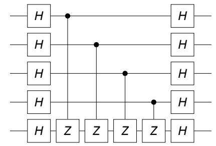
$$\hat{V}_v = \hat{X}_{v,1}\hat{X}_{v,2}\hat{X}_{v,3}\hat{X}_{v,4}$$







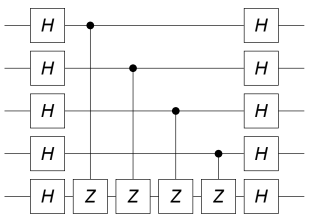


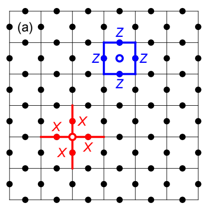


ERROR SYNDROME MEASUREMENTS

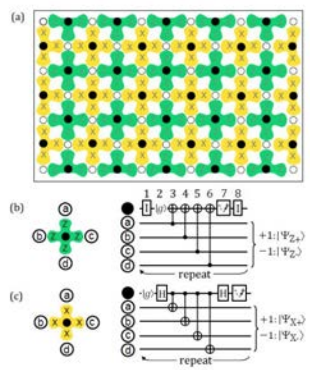
VERTEX OPERATORS

$$\hat{V}_v = \hat{X}_{v,1}\hat{X}_{v,2}\hat{X}_{v,3}\hat{X}_{v,4}$$





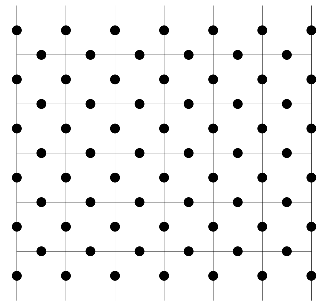
Fowler, Mariantoni, Martinis, & Cleland (Phys. Rev. A, 2012)



Planar Codes

Qubits Arranged on a Plane

Rough and Smooth Boundaries



Stabilizers

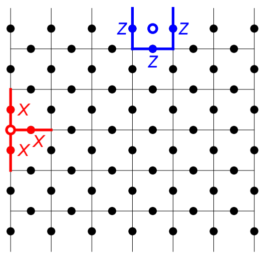
Plaquette & Vertex Operators

- Plaquette operators on the rough boundary

$$\hat{P}_p = \hat{Z}_{p,1}\hat{Z}_{p,2}\hat{Z}_{p,3}\cancel{\hat{Z}_{p,4}}$$

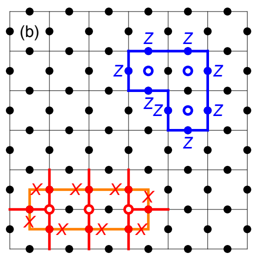
- Vertex operators on the smooth boundary

$$\hat{V}_v = \hat{X}_{v,1}\hat{X}_{v,2}\hat{X}_{v,3}\cancel{\hat{X}_{v,4}}$$

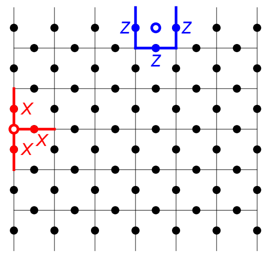


Multiplications of Stabilizers

Plaquette & Vertex Operators



$$\prod_p \hat{P}_p = \prod_v \hat{V}_v = \hat{I}$$

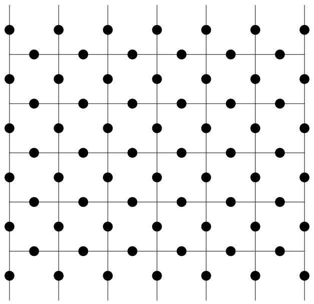


$$\cancel{\prod_p \hat{P}_p = \prod_v \hat{V}_v} = \hat{I}$$

How Many Stabilizers?

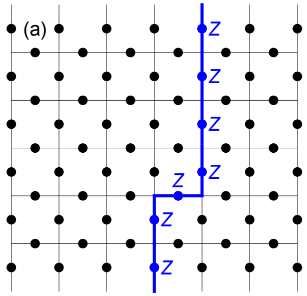
Dimension of the code space?

- There are L^2 plaquette operators.
- There are $(L+1)(L-1)$ vertex operators.
- Overall, there are $2L^2$-1 stabilizers.
- There are $L(L+1)+L(L-1)$ qubits (edges).
- The code can encode two logical qubits.



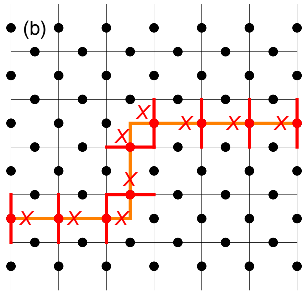
Where is one more compatible operator?

Where is a logical Z operator?



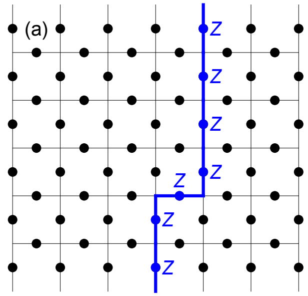
Where is one more compatible operator?

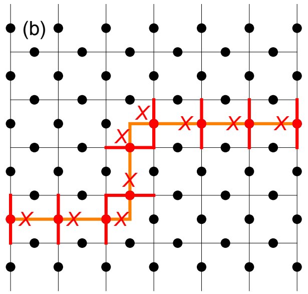
Where is a logical X operators?



Check!

Do they anti-commute?

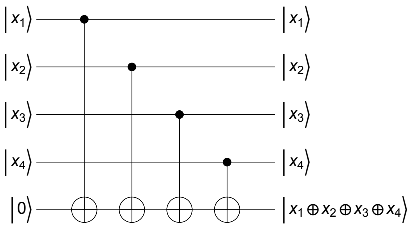


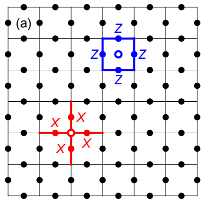


Error Syndrome Measurements

Plaquette Operators

$$\hat{P}_p = \hat{Z}_{p,1}\hat{Z}_{p,2}\hat{Z}_{p,3}\hat{Z}_{p,4}$$

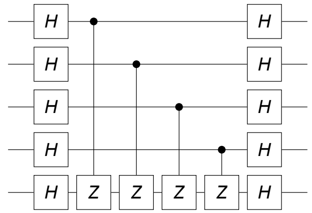


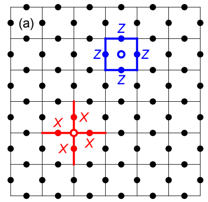


Error Syndrome Measurements

Vertex Operators

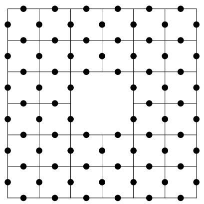
$$\hat{V}_v = \hat{X}_{v,1}\hat{X}_{v,2}\hat{X}_{v,3}\hat{X}_{v,4}$$

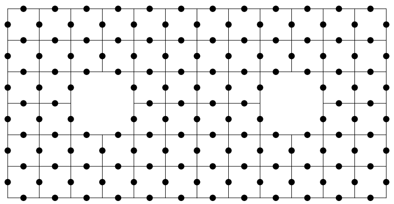




What if you want encode more logical qubits?

Can you figure out the dimension of the code space?



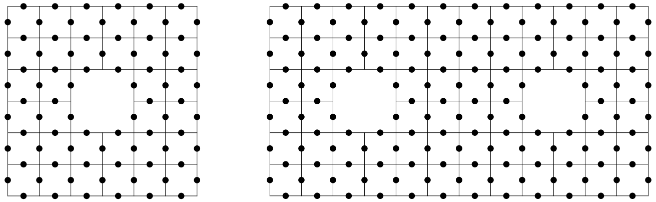


Topological Features

A Glimpse of Topological Quantum Computation

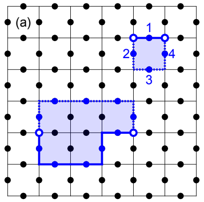
What if you want encode more logical qubits?

We have already seen one feature!



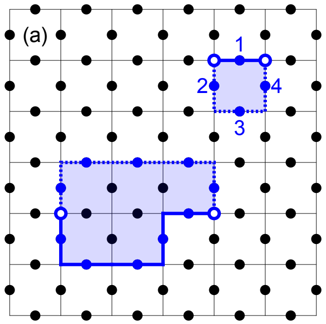
Correcting Phase-Flip Errors

- Suppose that a phase-flip occurs in qubit 1.
- This can be detected by measuring two vertex operators at the two ends of the edge.
- However, the syndrome is the same if the phase flips occur on edges 2, 3, and 4.
- Nevertheless, both case can be fixed by applying Z either on 1 or on 2, 3, 4.
- Here recall that $P=Z_1Z_2Z_3Z_4$ is a stabilizer.



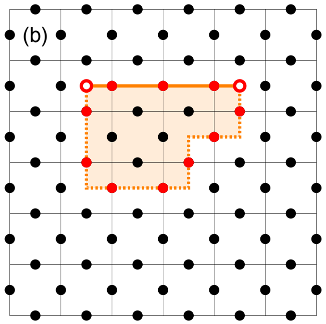
Correcting Phase-Flip Errors

Characterized by Vertex Defects



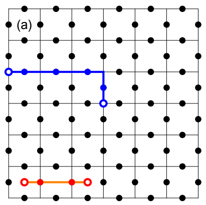
Correcting Bit-Flip Errors

Characterized by Plaquette Defects

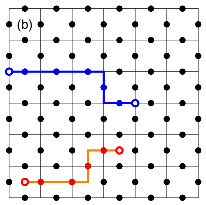


Topological Defects as “Particles”

Moving around the defects by applying Z or X



Before



After

Exchange of “Particles”

A. Yu. Kitaev (Ann. Phy., 2003; quant-ph/9707021)

- In the lower example, a vertex defect moves and returns to its original vertex without enclosing any other defect.
  It has no effect.
- In the upper example, the path along which the right vertex defect has traveled encloses a plaquette vertex.
  It causes a phase shift of -1.